Pure Mathematics P1 Mark scheme

Question	Scheme	Marks
1(a)	$y = 4x^3 - \frac{5}{x^2}$	
	$x^n \to x^{n-1}$ e.g. sight of x^2 **or** x^{-3} **or** $\frac{1}{x^3}$	M1
	$3 \times 4x^2$ **or** $-5 \times -2x^{-3}$ (o.e.) (Ignore + c for this mark)	A1
	$12x^2 + \frac{10}{x^3}$ **or** $12x^2 + 10x^{-3}$ <u>all on one line</u> and no + c	A1
		(3)
(b)	$x^n \to x^{n+1}$ e.g. sight of x^4 **or** x^{-1} **or** $\frac{1}{x^1}$	M1
	Do <u>not</u> award for integrating their answer to part (a) $4\frac{x^4}{4}$ **or** $-5 \times \frac{x^{-1}}{-1}$	A1
	For fully correct and simplified answer with + c <u>all on one line.</u> Allow $\Rightarrow$ Allow $x^4 + 5 \times \frac{1}{x} + c$ $\Rightarrow$ Allow $1x^4$ for x^4	A1
		(3)
		(6 marks)

Question	Scheme		Marks
2(a)	$3^{-1.5} = \dfrac{1}{3\sqrt{3}} \quad \left(\dfrac{\times\sqrt{3}}{\times\sqrt{3}}\right)$		M1
	$= \dfrac{\sqrt{3}}{9} \quad \text{so} \quad a = \dfrac{1}{9}$		A1
			(2)
	Alternative		
	$3^{-1.5} = a\sqrt{3} \Rightarrow a = \dfrac{3^{-1.5}}{3^{0.5}} = 3^{-1.5-0.5}$		M1
	$\Rightarrow a = 3^{-2} = \dfrac{1}{9}$		A1
(b)	$\left(2x^{\frac{1}{2}}\right)^3 = 2^3 x^{\frac{3}{2}}$	One correct power either 2^3 or $x^{\frac{3}{2}}$.	M1
	$\dfrac{8x^{\frac{3}{2}}}{4x^2} = 2x^{-\frac{1}{2}}$ **or** $\dfrac{2}{\sqrt{x}}$		dM1 A1
			(3)
			(5 marks)

Notes:

(a)

M1: Scored for a full attempt to write $3^{-1.5}$ in the form $a\sqrt{3}$ or, as an alternative, makes a the subject and attempts to combine the powers of 3

A1: For $a = \dfrac{1}{9}$ Note: A correct answer with no working scores full marks

(b)

M1: For an attempt to expand $\left(2x^{\frac{1}{2}}\right)^3$ Scored for one correct power either 2^3 or $x^{\frac{3}{2}}$.

$\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)\times\left(2x^{\frac{1}{2}}\right)$ on its own is not sufficient for this mark.

dM1: For dividing their coefficients of x **and** subtracting their powers of x. Dependent upon the previous M1

A1: Correct answer $2x^{-\frac{1}{2}}$ **or** $\dfrac{2}{\sqrt{x}}$

Question	Scheme		Marks
3	$y = -4x - 1$ $\Rightarrow (-4x-1)^2 + 5x^2 + 2x = 0$	Attempts to makes y the subject of the linear equation and substitutes into the other equation.	M1
	$21x^2 + 10x + 1 = 0$	Correct 3 term quadratic	A1
	$(7x+1)(3x+1) = 0 \Rightarrow (x =) -\frac{1}{7}, -\frac{1}{3}$	dM1: Solves a **3 term** quadratic by the usual rules A1: $(x =)\ -\frac{1}{7}, -\frac{1}{3}$	dM1A1
	$y = -\frac{3}{7}, \frac{1}{3}$	M1: Substitutes to find at least one y value A1: $y = -\frac{3}{7}, \frac{1}{3}$	M1 A1
			(6)
	Alternative		
	$x = -\frac{1}{4}y - \frac{1}{4}$ $\Rightarrow y^2 + 5\left(-\frac{1}{4}y - \frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}y - \frac{1}{4}\right) = 0$	Attempts to makes x the subject of the linear equation and substitutes into the other equation.	M1
	$\frac{21}{16}y^2 + \frac{1}{8}y - \frac{3}{16} = 0$ $(21y^2 + 2y - 3 = 0)$	Correct 3 term quadratic	A1
	$(7y+3)(3y-1) = 0 \Rightarrow (y =) -\frac{3}{7}, \frac{1}{3}$	Solves a **3 term** quadratic	dM1
		$(y =) -\frac{3}{7}, \frac{1}{3}$	A1
	$x = -\frac{1}{7}, -\frac{1}{3}$	Substitutes to find at least one x value.	M1
		$x = -\frac{1}{7}, -\frac{1}{3}$	A1
			(6)
			(6 marks)

Question	Scheme	Marks
4	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ o.e.	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times2\times(3 - c) = 0$ and so $c =$	dM1
	$c = 1$ **cso**	A1
		(5)
	Alternative 1A	
	Sets derivative "$4x + 8$" $= 4 \Rightarrow x =$	M1
	$x = -1$	A1
	Substitute $x = -1$ in $y = 2x^2 + 8x + 3$ ($\Rightarrow y = -3$)	dM1
	Substitute $x = -1$ and $y = -3$ in $y = 4x + c$ or into $(y + 3)=4(x + 1)$ and expand	dM1
	$c = 1$ or writing $y = 4x + 1$ **cso**	A1
		(5)
	Alternative 1B	
	Sets derivative "$4x+8$" $=4 \Rightarrow x =$,	M1
	$x = -1$	A1
	Substitute $x = -1$ in $2x^2 + 8x + 3 = 4x + c$	dM1
	Attempts to find value of c	dM1
	$c = 1$ or writing $y = 4x + 1$ **cso**	A1
		(5)
	Alternative 2	
	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	States that $b^2 - 4ac = 0$	dM1
	$4^2 - 4\times2\times(3 - c) = 0$ and so $c =$	dM1
	$c = 1$ **cso**	A1
		(5)
	Alternative 3	
	Sets $2x^2 + 8x + 3 = 4x + c$ and collects x terms together	M1
	Obtains $2x^2 + 4x + 3 - c = 0$ or equivalent	A1
	Uses $2(x+1)^2 - 2 + 3 - c = 0$ or equivalent	dM1
	Writes $-2 + 3 - c = 0$	dM1
	So $c = 1$ **cso**	A1
		(5)
		(5 marks)

Question 4 *continued*

Notes:

Method 1A

M1: Attempts to solve their $\frac{dy}{dx}=4$. They must reach $x=$... (Just differentiating is M0 A0).

A1: $x=-1$ (If this follows $\frac{dy}{dx}=4x+8$, then give M1 A1 by implication).

dM1: (Depends on previous M mark) Substitutes their x = -1 into f(x) or into "their f(x) from (b)" to find y.

dM1: (Depends on both previous M marks) Substitutes their x = -1 and their y = -3 values into $y = 4x + c$ to find c or uses equation of line is (y + "3") = 4(x + "1") and rearranges to $y = mx+c$

A1: c = 1 or allow for $y = 4x + 1$ cso.

Method 1B

M1A1: Exactly as in Method 1A above.

dM1: (Depends on previous M mark) Substitutes **their** x = -1 into $2x^2+8x+3 = 4x+c$

dM1: Attempts to find value of c then A1 as before.

Method 2

M1: Sets $2x^2+8x+3=4x+c$ and tries to collect x terms together.

A1: Collects terms e.g. $2x^2+4x+3-c=0$ or $-2x^2-4x-3+c=0$ or $2x^2+4x+3=c$ or even $2x^2+4x=c-3$. Allow "=0" to be missing on RHS.

dM1: Then use completion of square $2(x+1)^2-2+3-c=0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.

dM1: $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) ($x=-1$ has been used)

A1: c = 1 **cso**

Method 3

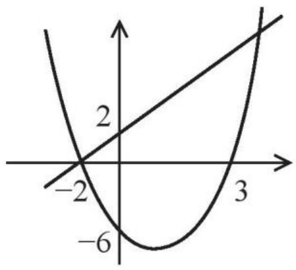
M1: Sets $2x^2+8x+3=4x+c$ and tries to collect x terms together. May be implied by $2x^2+8x+3-4x\pm c$ on one side.

A1: Collects terms e.g. $2x^2+4x+3-c=0$ or $-2x^2-4x-3+c=0$ or $2x^2+4x+3=c$ even $2x^2+4x=c-3$. Allow "=0" to be missing on RHS.

dM1: Then use completion of square $2(x+1)^2 - k + 3 - c = 0$ (Allow $2(x+1)^2 - k + 3 - c = 0$) where k is non zero. It is enough to give the correct or almost correct (with k) completion of the square.

dM1: $-2 + 3 - c = 0$ AND leading to a solution for c (Allow $-1 + 3 - c = 0$) (x = -1 has been used)

A1: c = 1 cso

Question		Marks
5(a)	 Straight line, positive gradient positive intercept	B1
	Curve 'U' shape anywhere	B1
	Correct y intercepts 2, -6	B1
	Correct x-intercepts of -2 and 3 with intersection shown at $(-2, 0)$	B1
		(4)
(b)	Finite region between line and curve shaded	B1
		(1)
(c)	$(x^2 - x - 6 < x + 2\ ) \Rightarrow x^2 - 2x - 8 < 0$	
	$(x-4)(\ x+2) < 0 \Rightarrow$ Line and curve intersect at $x = 4$ and $x = -2$	M1 A1
	$-2 < x < 4$	A1
		(3)
		(8 marks)

Notes:

(a) As scheme.

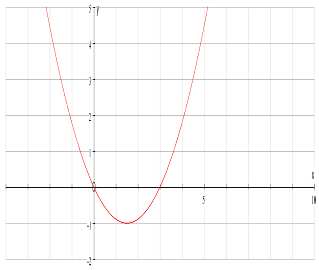
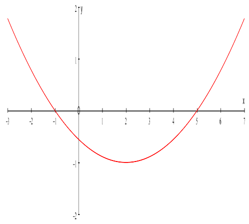
(b) As scheme.

(c)

M1: For a valid attempt to solve the equation $x^2 - 2x - 8 = 0$

A1: For $x = 4$ and $x = -2$

A1: $-2 < x < 4$

Question	Scheme		Marks
6(a)		Shape through (0, 0)	B1
		(3, 0)	B1
		(1.5, −1)	B1
			(3)
(b)		Shape , <u>not</u> through (0, 0)	B1
		Minimum in 4th quadrant	B1
		$(-p, 0)$ and $(6 - p, 0)$	B1
		$(3 - p, -1)$	B1
			(4)
			(7 marks)

Notes:

(a)

B1: U shaped parabola through origin.

B1: (3,0) stated or 3 labelled on x - axis (even (0,3) on x - axis).

B1: (1.5, -1) or equivalent e.g. (3/2, -1) labelled or stated and matching minimum point on the graph.

(b)

B1: Is for any translated curve to left or right or up or down not through origin

B1: Is for minimum in 4th quadrant and x intercepts to left and right of y axis (i.e. correct position).

B1: Coordinates stated or shown on x axis (Allow $(0 - p, 0)$ instead of $(-p, 0)$)

B1: Coordinates stated.

Note: If values are taken for p, then it is possible to give M1A1B0B0 even if there are several attempts. (In this case none of the curves should go through the origin for M1 and all minima should be in fourth quadrant and **all x intercepts need to be to left and right of y axis for A1**

Question	Scheme	Marks
7	$$\mathrm{f}(x)=\int\left(\frac{3}{8}x^2-10x^{-\frac{1}{2}}+1\right)\mathrm{d}x$$ $$x^n\to x^{n+1}\Rightarrow \mathrm{f}(x)=\frac{3}{8}\times\frac{x^3}{3}-10\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+x(+c)$$	M1 A1 A1
	Substitute $x = 4, y = 25 \Rightarrow 25 = 8 - 40 + 4 + c$ $\Rightarrow c =$	M1
	$$\mathrm{f}(x)=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$$	A1
		(5)
		(5 marks)

Notes:

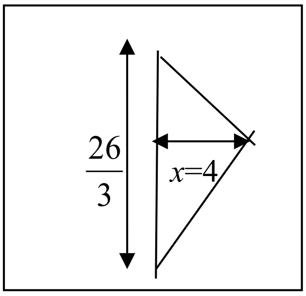
M1: Attempt to integrate $x^n \to x^{n+1}$

A1: Term in x^3 **or** term in $x^{\frac{1}{2}}$ correct, coefficient need not be simplified, no need for $+x$ nor +c

A1: ALL three terms correct, coefficients need not be simplified, no need for + c

M1: For using $x = 4$, $y = 25$ in their f(x) to form a linear equation in c and attempt to find c

A1: $=\frac{x^3}{8}-20x^{\frac{1}{2}}+x+53$ cao (all coefficients and powers must be simplified to give this answer- do not need a left hand side and if there is one it may be f(x) or y). Need full expression with 53. These marks need to be scored in part (a).

Question	Scheme	Marks
8(a)	$2x + 3y = 26 \Rightarrow 3y = 26 \pm 2x$ and attempt to find m from $y = mx + c$	M1
	$\left(\Rightarrow y = \frac{26}{3} - \frac{2}{3}x\right)$ so gradient $= -\frac{2}{3}$	A1
	Gradient of perpendicular $= \frac{-1}{\text{their gradient}}$ $\left(= \frac{3}{2}\right)$	M1
	Line goes through (0, 0) so $y = \frac{3}{2}x$	A1
		(4)
(b)	Solves their $y = \frac{3}{2}x$ with their $2x + 3y = 26$ to form equation in x or in y	M1
	Solves their equation in x or in y to obtain $x =$ **or** $y =$	dM1
	$x = 4$ or any equivalent e.g. $\frac{156}{39}$ **or** $y = 6$ o.a.e	A1
	$B = (0, \frac{26}{3})$ used or stated in (b)	B1
	 Area $= \frac{1}{2} \times "4" \times \frac{"26"}{3}$	dM1
	$= \frac{52}{3}$ (o.e. with integer numerator and denominator)	A1
		(6)
		(10 marks)

Notes:

(a)

M1: Complete method for finding gradient. (This may be implied by later correct answers.) e.g.
Rearranges $2x + 3y = 26 \Rightarrow y = mx + c$ so $m =$
Or finds coordinates of two points on line and finds gradient e.g.
$(13, 0)$ *and* $(1, 8)$ *so* $m = \frac{8-0}{1-13}$

A1: States or implies that gradient $= -\frac{2}{3}$ condone $= -\frac{2}{3}x$ if they continue correctly. Ignore errors in constant term in straight line equation.

M1: Uses $m_1 \times m_2 = -1$ to find the gradient of l_2. This can be implied by the use of $\frac{-1}{\text{their gradient}}$

A1: $y = \frac{3}{2}x$ or $2y - 3x = 0$ Allow $y = \frac{3}{2}x + 0$ Also accept $2y = 3x$, $y = \frac{39}{26}x$ or even $y - 0 = \frac{3}{2}(x - 0)$ and isw.

Question 8 notes *continued*

(b)

M1: Eliminates variable between their $y=\frac{3}{2}x$ and their (possibly rearranged) $2x+3y=26$ to form an equation in x or y. (They may have made errors in their rearrangement).

dM1: (Depends on previous M mark) Attempts to solve their equation to find the value of x or y

A1: $x = 4$ or equivalent or $y = 6$ or equivalent

B1: y coordinate of B is $\frac{26}{3}$ (stated or implied) - isw if written as ($\frac{26}{3}$, 0).

Must be used or stated in (b)

dM1: (Depends on previous M mark) Complete method to find area of triangle OBC (using their values of x and/or y at point C and their $\frac{26}{3}$)

A1: Cao $\frac{52}{3}$ or $\frac{104}{6}$ or $\frac{1352}{78}$ o.e

Alternative 1

Uses the area of a triangle formula ½×OB ×(x coordinate of C)

Alternative methods: Several Methods are shown below. The only mark which differs from Alternative 1 is the last M mark and its use in each case is described below:

Alternative 2

In 8(b) using $\frac{1}{2}\times BC\times OC$

dM1: Uses the area of a triangle formula ½×BC ×OC Also finds OC (=√52) and BC= ($\frac{4}{3}\sqrt{13}$)

Alternative 3

In 8(b) using $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$

dM1: States the area of a triangle formula $\frac{1}{2}\begin{vmatrix} 0 & 4 & 0 & 0 \\ 0 & 6 & \frac{26}{3} & 0 \end{vmatrix}$ or equivalent with their values

Alternative 4

In 8(b) using area of triangle OBX – area of triangle OCX where X is point (13, 0)

dM1: Uses the correct subtraction $\frac{1}{2}\times 13\times "\frac{26}{3}" - \frac{1}{2}\times 13\times "6"$

Alternative 5

In 8(b) using area = ½ (6 × 4) + ½ (4 × 8/3) drawing a line from C parallel to the x axis and dividing triangle into two right angled triangles

dM1: For correct method area = ½ (“6” × “ 4”) + ½ (“4” × [“26/3”-“6”])

Method 6 Uses calculus

dM1: $\int_0^4 "\frac{26}{3}" - \frac{2x}{3} - \frac{3x}{2}\,dx = \left[\frac{26}{3}x - \frac{x^2}{3} - \frac{3x^2}{4}\right]_0^4$

Question	Scheme	Marks
9(a)	Substitutes $x = 2$ into $y = 20 - 4\times 2 - \frac{18}{2}$ and gets 3	B1
	$\frac{dy}{dx} = -4 + \frac{18}{x^2}$	M1 A1
	Substitute $x = 2 \Rightarrow \frac{dy}{dx} = \left(\frac{1}{2}\right)$ then finds negative reciprocal (-2)	dM1
	States or uses $y - 3 = -2(x - 2)$ or $y = -2x + c$ with their (2, 3)	ddM1
	to deduce that $y = -2x + 7$	A1*
		(6)
(b)	Put $20 - 4x - \frac{18}{x} = -2x + 7$ and simplify to give $2x^2 - 13x + 18 = 0$ Or put $y = 20 - 4\left(\frac{7-y}{2}\right) - \frac{18}{\left(\frac{7-y}{2}\right)}$ to give $y^2 - y - 6 = 0$	M1 A1
	$(2x - 9)(x - 2) = 0$ so $x =$ **or** $(y - 3)(y + 2) = 0$ so $y =$	dM1
	$\left(\frac{9}{2}, -2\right)$	A1 A1
		(5)
		(11 marks)

Notes:

(a)

B1: Substitutes $x = 2$ into expression for y and gets 3 cao (must be in part (a) and must use curve equation – not line equation). This must be seen to be substituted.

M1: For an attempt to differentiate the negative power with x^{-1} to x^{-2}.

A1: Correct expression for $\frac{dy}{dx} = -4 + \frac{18}{x^2}$

dM1: Dependent on **first** M1 substitutes $x = 2$ into their derivative to obtain a numerical gradient and find negative reciprocal or states that $-2\times\frac{1}{2} = -1$

Alternative 1

dM1: Dependent on **first** M1. Finds equation of line using changed gradient (not their $\frac{1}{2}$ but $-\frac{1}{2}$ 2 or −2) e.g. $y - "3" = -"2"(x - 2)$ or $y = "-2"x + c$ and use of (2, "3") to find $c =$

A1*: cso. This is a given answer $y = -2x + 7$ obtained with no errors seen and equation should be stated.

Alternative 2 – checking given answer

dM1: Uses given equation of line and checks that (2, 3) lies on the line.

A1*: cso. This is a given answer $y = -2x + 7$ so statement that normal and line **have the same gradient** and **pass through the same point** must be stated.

Question 9 notes *continued*

(b)

M1: Equate the two given expressions, collect terms and simplify to a 3TQ. There may be sign errors when collecting terms but putting for example $20x - 4x^2 - 18 = -2x + 7$ is M0 here.

A1: Correct 3TQ = 0 (need = 0 for A mark) $2x^2 - 13x + 18 = 0$

dM1: Attempt to solve an appropriate quadratic by factorisation, use of formula, or completion of the square (see general instructions).

A1: $x = \frac{9}{2}$ o.e or $y = -2$ (allow second answers for this mark so ignore $x = 2$ or $y = 3$)

A1: Correct solutions only so both $x = \frac{9}{2}$, $y = -2$ or $\left(\frac{9}{2}, -2\right)$

If $x = 2$, $y = 3$ is included as an answer and point B is not identified then last mark is A0.
Answer only – with no working – send to review. The question stated 'use algebra'.

Question	Scheme		Marks
10(a)	$9^2 = 4^2 + 6^2 - 2\times4\times6\cos\alpha \Rightarrow \cos\alpha = ...$	Correct use of cosine rule leading to a value for cos α	M1
	$\cos\alpha = \dfrac{4^2+6^2-9^2}{2\times4\times6}\left(=-\dfrac{29}{48}=-0.604..\right)$		
	$\alpha = 2.22$ * cso		A1
			(2)
	Alternative		
	$XY^2 = 4^2 + 6^2 - 2\times4\times6\cos2.22 \Rightarrow XY^2 = ..$	Correct use of cosine rule leading to a value for XY^2	M1
	$XY = 9.00....$		A1
			(2)
(b)	$2\pi - 2.22(=4.06366......)$	$2\pi-2.22$ or $2\pi-2.2$ or awrt 4.06 (May be implied)	B1
	$\frac{1}{2}\times4^2\times"4.06"$	Correct method for major sector area. Allow $\pi-2.22$ for the major sector angle.	M1
	32.5	Awrt 32.5	A1
			(3)
	Alternative – Circle Minor – sector		
	$\pi\times4^2$	Correct expression for circle area	B1
	$\pi\times4^2 - \dfrac{1}{2}\times4^2\times2.22 = 32.5$	Correct method for circle - minor sector area	M1
	$= 32.5$	Awrt 32.5	A1
			(3)
(c)	Area of triangle = $\dfrac{1}{2}\times4\times6\times\sin2.22(=9.56)$	Correct expression for the area of triangle XYZ (allow 2.2 or awrt 2.22)	B1
	So area required = "9.56" + "32.5"	Their Triangle XYZ + part (b) or correct attempt at major sector (**Not** triangle ZXW)	M1
	Area of logo = 42.1 cm^2 or 42.0 cm^2	Awrt 42.1 or 42.0 (or just 42)	A1
			(3)
(d)	Arc length $=4\times4.06(=16.24)$ **or** $8\pi-4\times2.22$	M1: $4\times$*their*$(2\pi-2.22)$ **or** circumference – minor arc A1: Correct ft expression	M1 A1ft
	Perimeter = $ZY + WY$ + Arc Length	9 + 2 + Any Arc	M1
	Perimeter of logo = 27.2 or 27.3	Awrt 27.2 **or** awrt 27.3	A1
			(4)
			(12 marks)